PRODUCT MIX PROBLEMS

Problem 1.1

A garment manufacturer has a production line making two styles of shirts. Style I requires 200 grams of cotton thread, 300 grams of dacron thread, and 300 grams of linen thread. Style II requires 200 grams of cotton thread, 200 grams of dacron thread and 100 grams of linen thread. The manufacturer makes a net profit of Rs. 19.50 on Style 1, Rs. 15.90 on Style II. He has in hand an inventory of 24 kg of cotton thread, 26 kg of dacron thread and 22 kg of linen thread. His immediate problem is to determine a production schedule, given the current inventory to make a maximum profit. Formulate the LPP model.

Solution

|  |  |  |
| --- | --- | --- |
| Let | *x*1 = Number of Style I shirts, | |
| *x*2 = Number of Style II shirts. | |
| Since the objective is to maximise the profit, the objective function is given by — | |
| Maximize Z = 19.50*x*1 + 15.90*x*2 | |
| Subject to constraints: | |
| 200*x*1 + 200*x*2 ≤ 24,000 | [Maximum Qty of Cotton thread available] |
| 300*x*1 + 200*x*2 ≤ 26,000 | [Maximum Qty. of Dacron thread available] |
| 300*x*1 + 100*x*2 ≤ 22,000 | [Maximum Qty. of Lines thread available] |
| *x*1, *x*2 ≥ 0 | [Non-Negativity constraint) |

Problem 1.2

A firm makes two types of furniture: chairs and tables. The contribution for each product as calculated by the accounting department is Rs. 20 per chair and Rs. 30 per table. Both products are processed on three machines M1, M2 and M3. The time required by each product and total time available per week on each machine are as follows:

How should the manufacturer schedule his production in order to maximise contribution?

Solution

|  |  |  |
| --- | --- | --- |
| Let | *x*1 = Number of chairs to be produced | |
| *x*1 = Number of tables to be produced | |
| Since the objective is to maximise the profit, the objective function is given by — | |
| Maximise Z = 20*x*1 + 30*x*2 | |
| Subject to constraints: | |
| 3*x*1 + 3*x*2 ≤ 36 | (Total time of machine M1) |
| 5*x*1 + 2*x*2 ≤ 50 | (Total time of machine M2) |
| 2*x*1 + 6*x*2 ≤ 60 | (Total time of machine M3) |
| *x*1, *x*2 ≥ 0 | (Non-negativity constraint) |

Problem 1.3

The ABC manufacturing company can make two products P1 and P2. Each of the products requires time on a cutting machine and a finishing machine. Relevant data are:

|  |  |  |
| --- | --- | --- |
|  | *Product* | |
|  | *P1* | *P2* |
| Cutting Hours (per unit) | 2 | 1 |
| Finishing Hours (per unit) | 3 | 3 |
| Profit (Rs. per unit) | 6 | 4 |
| Maximum sales (unit per week) |  | 200 |

The number of cutting hours available per week is 390 and the number of finishing hours available per week is 810. How much should be produced of each product in order to achieve maximum profit for the company?

Solution

|  |  |  |
| --- | --- | --- |
| Let | *x*1 = Number of Product P1 to be produced | |
| *x*1 = Number of Product P2 to be produced | |
| Since the objective is to maximise profit, the objective function is given by — | |
| Maximise Z = 6*x*1 + 4*x*2 | |
| Subject to constraints: | |
| 2*x*1 + *x*2 ≤ 390 | (Availability of cutting hours) |
| 3*x*1 + 3*x*1 ≤ 810 | (Availability of finishing hours) |
| *x*2 ≤ 200 | (Maximum sales) |
| *x*1, *x*2 ≥ 0 | (Non-negativity constraint) |

Problem 1.4

A company makes two kinds of leather belts. Belt A is a high quality belt, and belt B is of lower quality. The respective profits are Re. 0.40 and Re. 0.30 per belt. Each belt of type A requires twice as much time as a belt of type B, and if all belts were of type B, the company could make 1,000 per day. The supply of leather is sufficient for only 800 belts per day (both A and B combined). Belt A requires a fancy buckle, and only 400 per day are available. There are only 700 buckles a day available for belt B.

What should be the daily production of each type of belt? Formulate the linear programming problem.

Solution

|  |  |  |
| --- | --- | --- |
| Let | *x*1 = Number of Belt A to be produced | |
| *x*2 = Number of Belt B to be produced | |
| Since the objective is to maximise the profit, the objective function is given by — | |
| Maximise Z = .40*x*1 + .30*x*2 | |
| Subject to constraints: | |
| 2*x*1 + *x*2 ≤ 1000 | (Total availability of time) |
| *x*1 + *x*2 ≤ 800 | (Total availability of leather) |
| *x*1 ≤ 400 | (Availability of buckles for belt A) |
| *x*2 ≤ 700 | (Availability of buckles for belt B) |
| *x*1, *x*2 ≥ 0 | (Non-negativity constraint) |

Problem 1.5

Mr. Jain, the marketing manager of ABC Typewriter Company is trying to decide on how to allocate his salesmen to the Company’s three primary markets. Market-1 is an urban area and the salesmen can sell, on an average 40 typewriters a week. Salesmen in the other two markets can sell, on an average, 36 and 25 typewriters per week, respectively. For the coming week, 3 of the salesmen will be on vacation, leaving only 12 men available for duty. Also because of the lack of company cars, maximum of 5 salesmen can be allocated to market area 1. The selling expenses per week per salesman in each area are Rs. 800 per week for area 1, Rs. 700 per week for area 2, and Rs. 500 per week for area 3. The budget for the next week is Rs. 7500. The profit margin per typewriter is Rs. 150.

Formulate a linear programming model to determine how many salesmen should be assigned to each area in order to maximise profits.

Solution

|  |  |  |
| --- | --- | --- |
| Let | *x*1 = Number of salesmen to be allocated to area 1 | |
| *x*2 = Number of salesmen to be allocated to area 2 | |
| *x*3 = Number of salesmen to be allocated to Area 3 | |
| Since the objective is to maximise the profit, the objective function is given by — | |
| Maximise Z = 150 (40*x*1) + 150 (36*x*2) + 150 (25*x*3)                    = 6,000*x*1 + 5,400*x*2 + 3,750*x*3 | |
| Subject to constraint: |  |
| *x*1 + *x*2 + *x*3 ≤ 12 | (Availability of total salesmen) |
| *x*1 ≤ 5 | (Maximum salesmen for area 1) |
| 800*x*1 + 700*x*2 + 500*x*3 ≤ 7500 | (Budgeted expenses) |
| *x*1, *x*2, *x*3 ≥ 0 | (Non-negativity constraint) |

Problem 1.6

An animal feed company must produce 200 kg of a mixture consisting of ingredients X1, and X2 daily. X1 cost Rs. 3 per kg and X2 Rs. 8 per kg. Not more than 80 kg of X1 can be used, and at least 60 kg of X2 must be used. Find how much of each ingredient should be used if the company wants to minimise cost.

Solution

|  |  |  |
| --- | --- | --- |
| Let | *x*1 = kg of ingredient *X*1 to be used | |
| *x*1 = kg of ingredient *X*2 to be used | |
| Since the objective is to minimise the cost, the objective function is given by — | |
| Minimise Z = 3*x*1 + 8*x*2 | |
| Subject to constraints: | |
| *x*1 + *x*2 = 200 | (Total mixture to be produced) |
| *x*1 ≤ 80 | (Maximum use of *x*1) |
| x2 ≥ 60 | (Minimum use of *x*2) |
| *x*1 ≥ 0 | (Non-negativity constraint) |

Problem 1.7

The ABC Printing Company is facing a tight financial squeeze and is attempting to cut costs wherever possible. At present it has only one printing contract and, luckily, the book is selling well in both the hardcover and paperback editions. It has just received a request to print more copies of this book in either the hardcover or paperback form. Printing cost for hardcover books is Rs. 600 per 100 while printing cost for paperback is only Rs. 500 per 100. Although the company is attempting to economise, it does not wish to lay off any employees. Therefore, it feels obliged to run its two printing presses at least 80 and 60 hours per week, respectively. Press 1 can produce 100 hardcover books in 2 hours or 100 paperback books in 1 hours. Press II can produce 100 hardcover books in 1 hours or 100 paperback books in 2 hours. Determine how many books of each type should be printed in order to minimise cost.

Solution

|  |  |  |
| --- | --- | --- |
| Let | *x*1 = Number of hardcover books (per 100) to be produced | |
| *x*2 = Number of paper back books (per 100) to be produced | |
| Since the objective is to minimise the cost, the objective function is given by — | |
| Minimise Z = 600*x*1 + 500*x*2 | |
| Subject to constraints: | |
| 2*x*1 + *x*2 ≤ 80 | (Minimum running of Press I) |
| *x*1 + 2*x*2 ≤ 60 | (Minimum running of Press II) |
| *x*1, *x*2 ≥ 0(Non-negativity constraint) | |

Problem 1.8

A medical scientist claims to have found a cure for the common cold that consists of three drugs called K, S and H. His results indicate that the minimum daily adult dosage for effective treatment is 10 mg. of drug K, 6 mg. of drug S, and 8 mg. of drug H. Two substances are readily available for preparing pills or drugs. Each unit of substance A contains 6 mg., 1 mg. and 2 mg. of drugs K, S and H respectively, and each unit of substance B contains 2 mg, 3 mg, and 2 mg., of the same drugs. Substance A costs Rs. 3 per unit and substance B costs Rs. 5 per unit.

Find the least-cost combination of the two substances that will yield a pill designed to contain the minimum daily recommended adult dosage.

Solution

|  |  |  |
| --- | --- | --- |
| Let | *x*1 = Substance A | |
| *x*2 = Substance B | |
| Minimise Z = 3*x*1 + 5*x*2 | |
| Subject to constraints: | |
| 6*x*1 + 2*x*2 = 10 | (Requirement of drug K) |
| *x*1 + 3*x*2 = 6 | (Requirement of drug S) |
| 2*x*2 + 2*x*2 = 8 | (Requirement of drug H) |
| *x*1, *x*2 ≥ 0 | (Non-negativity constraint) |

Problem 1.9

A publisher of textbooks is in the process of presenting a new book to the market. The book may be bound by either cloth or hard paper. Each cloth bound book sold contributes Rs. 24, and each paper-bound book contributes Rs. 23. It takes 10 minutes to bind a cloth cover, and 9 minutes to bind a paperback. The total available time for binding is 800 hours. After considerable market survey, it is predicted that the cloth-cover sales will exceed at least 10,000 copies, but the paperback sales will be not more than 6,000 copies. Formulate the problem as a LP problem.

Solution

|  |  |  |
| --- | --- | --- |
| Let | *x*1 = Number of books bound by cloth | |
| *x*2 = Number of books bound by hard paper | |
| Since the objective is to maximise the profit, the objective function is given by — | |
| Maximise Z = 24*x*1 + 23*x*2 | |
| Subject to constraints: | |
| 10*x*1 + 9*x*2 ≤ 48,000 | (Total time available in minutes) |
| *x*1 ≥ 10,000 | (Minimum sales of cloth-cover books) |
| *x*2 ≤ 6,000 | (Maximum sales of hard paper books) |
| *x*2 ≥ 0 | (Non-negativity constraint) |

Problem 1.10

A company manufacturing television sets and radios has four major departments: chasis, cabinet, assembly and final testing. Monthly capacities are as follows:

The contribution of television is Rs. 150 each and the contribution of radio is Rs. 250 each. Assuming that the company can sell any quantity of either product, determine the optimal combination of output.

Solution

|  |  |  |
| --- | --- | --- |
| Let | *x*1 = Number of televisions to be produced | |
| *x*2 = Number of radio to be produced | |
| Since the objective is to maximise the profit, the objective function is given by — | |
| Maximise Z = 150*x*1 + 250*x*2 | |
| Subject to constraints: | |
| 3*x*1 + *x*2 ≤ 4,500 | (Maximum capacity in chasis department) |
| 8*x*1 + *x*2 ≤ 8,000 | (Maximum capacity in cabinet department) |
| 2*x*1 + *x*2 ≤ 4,000 | (Maximum capacity in assembly department) |
| 3*x*1 + *x*2 ≤ 9,000 | (Maximum capacity in testing department) |
| *x*1, *x*2 ≥ 0 | (Non-negativity) |

Problem 1.11

A timber company cuts raw timber-oak and pine logs into wooden boards. Two steps are required to produce boards from logs. The first step involves removing the bark from the logs. Two hours are required to remove bark from 1,000 feet of oak logs and three hours per 1,000 feet of pine logs. After the logs have been debarked, they must be cut into boards. It takes 2.4 hours for cutting 1,000 feet of oak logs into boards and 1.2 hours for 1,000 feet of pine logs. The bark removing machines can operate up to 60 hours per week, while the cutting machine are limited to 48 hours per week. The company can buy a maximum of 18,000 feet of raw oak logs and 12,000 feet of raw pine logs each week. The profit per 1,000 feet of processed logs is Rs. 1,800 and Rs. 1,200 for oak and pine logs, respectively. Solve the problem to determine how many feet of each type of log should be processed each week in order to maximise profit.

Solution

|  |  |  |
| --- | --- | --- |
| Let | *x*1 = Feet of logs (in 1000 feets) of oak to be produced | |
| *x*2 = Feet of logs (in 1000 feets) of pine to be produced | |
| Since the objective is to maximise the profit, the objective function is given by — | |
| Maximise Z = 1800*x*1 + 1200*x*2 | |
| Subject to constraints: | |
| 2*x*1 + 3*x*2 ≤ 60 | (Maximum machine hours for bark-removing) |
| 2.4*x*1 + 1.2*x*2 ≤ 48 | (Maximum machine hours for cutting) |
| *x*1 ≤ 18 | (Maximum availability of raw oak) |
| *x*2 ≤ 12 | (Maximum availability of raw pine) |
| *x*1, *x*2 ≥ 0 | (Non-negativity constraint) |

Problem 1.12

Upon completing the construction of his house, Mr. Sharma discovers that 100 square feet of plywood scrap and 80 square feet of white pine scrap are in usable form for the construction of tables and book cases. It takes 16 square feet of plywood and 16 square feet of white pine to construct a book case. It takes 20 square feet of plywood and 20 square feet of white pine to construct a table. By selling the finished products to a local furniture store, Mr. Sharma can realise a profit of Rs. 25 on each table and Rs. 20 on each book-case. How can he most profitably use the left-over wood?

Solution

|  |  |  |
| --- | --- | --- |
| Let | *x*1 = Number of tables to be produced | |
| *x*1 = Number of book cases to be produced | |
| Since the objective is to maximise the profit, the objective function is given by — | |
| Maximise Z = 25*x*1 + 20*x*2 | |
| Subject to constraints: | |
| 20*x*1 + 16*x*2 ≤ 100 | (Maximum plywood scrap available) |
| 20*x*1 + 16*x*2 ≤ 80 | (Maximum white pine scrap available) |
| *x*1, *x*2 ≥ 0 | (Non-negativity constraint) |

Problem 1.13

A rubber company is engaged in producing three different kinds of tyres A, B and C. These three different tyres are produced at the company’s two different plants with different production capacities. In a normal 8 hours working day, Plant 1 produces 50, 100 and 100 tyres of type A, B and C, respectively. Plant 2, produces 60, 60 and 200 tyres of type A, B and C, respectively. The monthly demand for type A, B and C is 2,500, 3,000 and 7,000 units, respectively. The daily cost of operation of Plant 1 and Plant 2 is Rs. 2,500 and Rs. 3,500, respectively. Form LP Model to determine the minimum number of days of operation per month at two different plants to minimise the total cost while meeting the demand.

Solution

|  |  |  |
| --- | --- | --- |
| Let | *x*1 = No of days of operation in Plant 1 | |
| *x*2 = No of days of operation in Plant 2 | |
| Since the objective is to minimise the cost, the objective function is given by — | |
| Minimise Z = 2500*x*1 + 3500*x*2 | |
| Subject to constraints: | |
| 50*x*1 + 60*x*2 = 2,500 | (Requirement of type A) |
| 100*x*1 + 60*x*2 = 3000 | (Requirement of type B) |
| 100*x*1 + 200*x*2 = 7000 | (Requirement of type C) |
| *x*1, *x*2 ≥ 0 | (Non-negativity constraint) |

Problem 1.14

Two products A and B are to be manufactured. A single unit of product A requires 2.4 minutes of punch press time and 5 minutes of assembly time. The profit for product A is Rs. 0.60 per unit. A single unit of product B requires 3 minutes of punch press time and 2.5 minutes of welding time. The profit for product B is Rs. 0.70 per unit. The capacity of the punch press department available for these products is 1,200 minutes/week. The welding department has an idle capacity of 600 minutes/week and assembly department has 1,500 minutes/week.

1. Formulate the problem as linear programming problem.
2. Determine the quantities of products A and B so that total profit is maximised.

Solution

|  |  |  |
| --- | --- | --- |
| Let | *x*1 = Number of product A to be manufactured | |
| *x*2 = Number of product B to be manufactured | |
| Since the objective is to maximise the profit, the objective function is given by — | |
| Maximise Z = .60*x*1 + .70*x*2 | |
| Subject to constraints: | |
| 2.4*x*1 + 3*x*2 ≤ 1200 | (Availability of punch press time) |
| 5*x*1 ≤ 1500 | (Availability of assembly time) |
| 2.5*x*2 ≤ 600 | (Availability of welding time) |
| *x*1, *x*2 ≥ 0 | (Non-negativity constraint) |

Problem 1.15

The XYZ company during the festival season combines two factors A and B to form a gift pack which must weigh 5 kg. At least 2 kg. of A and not more than 4 kg. of B should be used. The net profit contribution to the company is Rs. 5 per kg. for A and Rs. 6 per kg. for B. Formulate LP Model to find the optimal factor mix.

Solution

|  |  |  |
| --- | --- | --- |
| Let | *x*1 = Kgs of Product A | |
| *x*2 = Kgs of Product B | |
| Since the objective is to maximise the profit, the objective function is given by — | |
| Maximise Z = 5*x*1 + 6*x*2 | |
| Subject to constraints: | |
| *x*1 + *x*2 = 5 | (Total weight of the pack) |
| *x*1 ≥ 2 | (Minimum requirement of A) |
| *x*2 ≤ 4 | (Maximum requirement of B) |
| *x*1, *x*2 ≥ 0 | (Non-negativity constraint) |

Problem 1.16

An advertising agency wishes to reach two types of audiences; customers with annual income of more than Rs. 15,000 (target audience A) and customers with annual income of less than Rs. 15,000 (target audience B). The total advertising budget is Rs. 2,00,000. One programme of TV advertising costs Rs. 50,000 and one programme of radio advertising costs Rs. 20,000. For contract reasons, at least 3 programmes have to be on TV and the number of radio programme must be limited to 5. Surveys indicate that a single TV programme reaches 4,50,000 customers in target audience A and 50,000 in the target audience B. One radio programme reaches 20,000 in target audience A and 80,000 in the target audience B. Determine the media-mix to maximise the total reach.

Solution

**Effective Exposure**

|  |  |  |
| --- | --- | --- |
| Let | *x*1 = Number of advertisement on Television | |
| *x*2 = Number of advertisement on Radio | |
| Since the objective is to maximize the total audience, the objective function is given by — | |
| Maximise Z = 5,00,000*x*1 + 1,00,000*x*2 | |
| Subject to constraints: | |
| 50,000*x*1 + 20,000*x*2 ≤ 2,00,000 | (Total amount available) |
| *x*1 ≥ 3 | (Minimum advertisement on TV) |
| *x*2 ≤ 5 | (Maximum advertisement on Radio) |
| *x*2 ≥ 0 | (Non-negativity constraint) |

Problem 1.17

POR Feed Company markets two feed mixes for cattle. The first mix, Fertilex, requires at least twice as much wheat as barely. The second mix, Multiplex, requires at least twice as much barley as wheat. Wheat costs Rs. 1.50 per kg., and only 1,000 kg. are available this month. Barley costs Rs. 1.25 per kg. and 1,200 kg. are available this month. Fertilex sells for Rs. 1.80 per kg. up to 99 kg. and each additional kg. over 99 sells for Rs. 1.65, Multiplex sells at Rs. 1.70 per kg. up to 99 kg. and each additional kg. over 99 kg. sells for Rs. 1.55 Bharat Farms will buy any and all amounts of both mixes of POR Feed Company. Set up the linear programming problem to determine the product mix that results in maximum profits.

Solution

|  |  |
| --- | --- |
| Let | *x*1 = Quantity of wheat to be mixed in Fertilex upto 99 kg |
| *x*2 = Quantity of barley to be mixed in Fertilex upto 99 kg |
| *x*3 = Quantity of wheat to be mixed in Fertilex for above 99 kg |
| *x*4 = Quantity of barley to be mixed in Fertilex for above 99 kg |
| *x*5 = Quantity of wheat to be mixed in Multiplex upto 99 kg |
| *x*6 = Quantity of barley to be mixed in Multiplex upto 99 kg |
| *x*7 = Quantity of wheat to be mixed in Multiplex for above 99 kg |
| *x*8 = Quantity of barley to be mixed in Multiplex for above 99 kg |

**Calculation of contribution per unit**

|  |  |  |
| --- | --- | --- |
|  | Since the objective is to maximise the profits, the objective function is given by — | |
| Maximise Z = .30*x*1 + .55*x*2 + .15*x*3 + .40*x*4 + .20*x*5 + .45*x*6 + .05*x*7 + .30*x*8 | |
| Subject to constraints: | |
| *x*1 ≥ 2*x*2 | [Quantity mix constraint] |
| *x*3 ≥ 2*x*4 | [Quantity mix constraint] |
| *x*1 + *x*2 ≤ 99 | [Sales constraint] |
| *x*6 ≥ 2*x*5 | [Quantity mix constraint] |
| *x*8 ≥ 2*x*7 | [Quantity mix constraint] |
| *x*5 + *x*6 ≤ 99 | [Sales constraint] |
| *x*1 + *x*3 + *x*5 + *x*7 ≤ 1000 | [Wheat supply constraint] |
| *x*2 + *x*4 + *x*6 + *x*8 ≤ 1,200 | [Barley supply constraint] |
| *x*1, *x*2, *x*3, *x*4, *x*5, *x*6, *x*7, *x*8 ≥ 0 | [Non-negativity constraint] |

Problem 1.18

A company has three operational departments (weaving, processing and packing) with capacity to produce three different types of clothes namely suitings, shirtings and woollens yielding the profit Rs. 2. Rs. 4 and Rs. 3 per meter respectively. One-meter suiting requires 3 minutes in weaving, 2 minutes in processing and 1 minutes in packing. Similarly one meter of shirting requires 4 minutes in weaving. 1 minute in processing and 3 minutes in packing while one meter woollen requires 3 minutes in each department. In a week, total run time of each department is 60, 40 and 80 hours for weaving, processing and packing departments respectively.

Formulate the linear programming problem to find the product mix to maximise the profit.

Solution

Let *x*1, *x*2 and *x*3 denote the number of meters produced of suitings, shirtings and woollens respectively. The given data can be tabulated as below:

Since the objective of the company is to find the product mix to maximise the profit, the objective function is given by —

Maximise *Z* = 2*x*1 + 4*x*2 + 3*x*3

Subject to the constraints:

|  |  |  |
| --- | --- | --- |
|  | 3*x*1 + 4*x*2 + 3*x*3 ≤ 3600 | (Maximum weaving time) |
| 2*x*1 + *x*2 + 3*x*3 ≤ 2400 | (Maximum processing time) |
| *x*1 + 3*x*2 + 3*x*3 ≤ 4800 | (Maximum packing time) |
| where *x*1, *x*2 and *x*3 ≥ 0 | (Non-negativity constraint) |

Problem 1.19

For XYZ Limited the following data are relevant to its products L and P:

|  | ***Product L*** | ***Product P*** |
| --- | --- | --- |
| ***Per unit*** | ***Rs*** | ***Rs*** |
| Selling price | 200.00 | 240.00 |
| Costs: |  |  |
| Direct materials | 45.00 | 50.00 |
| Direct wages: |  |  |
| Department: 1 | 16.00 | 20.00 |
| 2 | 22.50 | 13.50 |
| 3 | 10.00 | 30.00 |
| Variable overhead | 6.50 | 11.50 |

Fixed overhead is budgeted at Rs 275,000 per annum.

Relevant data for each department are:

In the present environment, it is not possible to engage any more employees.

Show mathematically the objective functions and the constraints.

Solution

**Calculation of contribution per unit of product ‘L’ and product ‘P’**

**Calculation of Maximum hours and hours per unit**

Let, *x*1 = Number of units of Product L to be produced

*x*2 = Number of units of Product P to be produced.

Since the objective is to maximise contribution per product, the objective function is given by —

Maximise (Total contribution)

      Z = 100*x*1 + 115*x*2

Subject to the constraints:

|  |  |  |
| --- | --- | --- |
|  | 8*x*1 + 10*x*2 ≤ 800 | (Maximum hours of Deptt. 1 constraints) |
| 10*x*1 + 6*x*2 ≤ 600 | (Maximum hours of Deptt. 2 constraints) |
| 4*x*1 + 12*x*2 ≤ 720 | (Maximum hours of Deptt. 3 constraints) |
| *x*1 ≥ 0, *x*2 ≥ 0 | (Non-negativity constraints) |

Problem 1.20

A Company makes three products X, Y and Z which flow through three departments: Drill, Lathe and Assembly. The hours of department time required by each of the products, the hours available in each of the departments, the marginal contribution of each of the products, and the estimated incremental cost of idle time per hour are given in the following table:

Determine the optimum product-mix.

Solution

Let *x*1, *x*2, *x*3 be the number of units of product X, Y and Z respectively and S1, S2 and S3 be Idle Hours of Drill, Lathe and Assembly departments respectively.

|  |  |  |
| --- | --- | --- |
|  | Maximize (Total contribution) | |
| Z = 90*x*1 + 150*x*2 + 200*x*3 – 85*s*1 – 115*s*2 – 130*s*3 | |
| Subject to 3*x*1 + 6*x*2 + 7*x*3 + *s*1 = 180 | (Maximum drill hours) |
| 3*x*1 + 5*x*2 + 4*x*3 + *s*2 = 240 | (Maximum lathe hours) |
| 8*x*1 + 10*x*2 + 12*x*3 + *s*3 = 860 | (Maximum assembly hours) |
| *x*1, *x*2, *x*3, *s*1, *s*2, *s*3 ≥ 0 | (Non-negativity constraints) |

Problem 1.21

A company produces three types of parts for automatic washing machines. It purchases castings of the parts from a local foundry and then finishes the parts on drilling, shaping, and polishing machines. The selling prices of parts A, B, and C respectively are Rs. 8, Rs. 10 and Rs. 14. All parts made can be sold. Castings for parts A, B and C, respectively, cost Rs. 5, Rs. 6 and Rs. 10. The company possesses only one of each type of machine. Costs per hour to run each of the three machines are Rs. 20 for drilling, Rs. 30 for shaping, and Rs. 30 for polishing. The capacities (parts per hour) for each part on each machine are shown in the following table:

The manager of the company wants to know how many of each type to produce per hour in order to maximize profit for the hour’s run. Formulate the above problem as a linear programming problem.

Solution

**Calculation of profit per unit**

Let *x*1, *x*2 and *x*3 denote the number of three type of parts to be produced per hour. Since the objective is to maximize the profit, the objective function (Z) is given by -

Maximize Z = .25*x*1 + *x*2 + .95*x*3

Subject to the constraints:

|  |  |
| --- | --- |
|  | (Drilling Constraint) |
|  | (Shaping Constraint) |
|  | (Polishing Constraint) |
| *x*1, *x*2, *x*3 ≥ 0. | (Non-Negativity Constraint) |

Problem 1.22

ABC Paints Company manufactures three grades of paints — Venus, Diana and Aurora. The Plant operates on a three shift basis and the following data are available from the production records:

There are no limitations on other resources. The particulars of sale forecasts and estimated contribution to overheads and profits are given below:

Due to commitments already made a minimum of 200 kilolitres per month of Aurora has to be necessarily supplied during the next year.

Just as the company was able to finalise the monthly production programme for the next 12 months, an offer was received from a nearby competitor for hiring 40 machine shifts per month of milling capacity for grinding Diana paint, that can be spared for at least a year. However, due to additional handling and profit margin of the competitor involved, by using this facility, the contribution from Diana will get reduced by 1000 kilolitre. Formulate (do not solve) the linear programming model for determining the monthly production programme to maximize contribution.

Solution

Let *x*1, *x*2 and *x*4 be the quantity in kilolitres of Venus, Diana and Aurora grade paints respectively that the company decides to manufacture. Also, let *x*3 be the quantity in kilolitres of Diana grade to be manufactured by using hired facilities for milling.

The given information can now be presented in appropriate mathematical form as follows:

|  |  |  |
| --- | --- | --- |
|  | Maximum (Total Profit) | |
| Z = 4,000*x*1 + 3,500*x*2 + (3,500 – 1,000) *x*3 + 2,000*x*4 | |
| Subject to the constraints: | |
| 0.30*x*1 + 0.15*x*2 + 0.15*x*3 + 0.75*x*4 ≤ 600 (tonnes) | (special additive constraint) |
| (machine shifts) | (own milling facility constraint) |
|  | (hired milling facility constraint) |
| (shifts) | (packing constraint) |

|  |  |  |
| --- | --- | --- |
|  | *x*1 ≤ 100 | (Venus Maximum Sales constraint) |
| *x*2 + *x*3 ≤ 400 | (Diana Maximum Sales constraint) |
| *x*4 ≤ 600 | (Aurora Maximum Sales constraint) |
| *x*4 ≥ 200 | (Aurora Maximum Sales constraint) |
| *x*1, *x*2, *x*3, *x*4 ≥ 0. | (Non-Negativity constraint) |

Problem 1.23

Consider a company that must produce two products over a production period of three months of duration. The company can pay for materials and labour from two sources: company funds and borrowed funds.

The firm faces three decisions:

1. How many units should it produce of Product 1?
2. How many units should it produce of Product 2?
3. How much money should it borrow to support the production of the two products?

In making these decisions, the firm wishes to maximise the profit contribution subject to the conditions stated below:

1. Since the company’s products are enjoying a seller’s market, it can sell as many units as it can produce. The company would therefore like to produce as many units as it can produce. The company would therefore like to produce as many units as possible subject to production capacity and financial constraints. The capacity constraints, together with cost and price data, are given in [Table-1](http://my.safaribooksonline.com/9789332512085/ch1sec8_xhtml#Para_75).

**Table 1:** Capacity, Price and Cost Data

1. The available company funds during the production period will be Rs. 3 lakhs.
2. A bank will give loans upto Rs. 2 lakhs per production period at an interest rate of 20 percent per annum provided the company’s acid (quick) test ratio is at least 1 to 1 while the loan is outstanding. Take a simplified acid-test ration given by
3. Also make sure that the needed funds are made available for meeting the production costs.

Formulate the above as a Linear Programming Problem.

Solution

|  |  |  |
| --- | --- | --- |
|  | Let | *x*1 = No. of units of product 1 produced |
| *x*2 = No. of units of product 2 produced |
| *x*3 = Amount of money borrowed. |

The profit contribution per unit of each product is given by the selling price minus the variable cost of production. Total Profit may be computed by summing up the profits from producing the two products minus the cost associated with borrowed funds (if any):

The objective function is thus stated as

|  |  |  |
| --- | --- | --- |
|  | Maximise Z = (14 – 10)*x*1 + (11 – 8)*x*2 – 0.05*x*3 | |
| = 4*x*1 **+** 3*x*2 *–* 0.05*x*3 | (Note that the interest rate is 20% per annum, hence 5% for a period of three months) |

subject to the following constraints:

The production capacity constraints for each department, as given by [table 1](http://my.safaribooksonline.com/9789332512085/ch1sec8_xhtml#Para_75) are

|  |  |  |
| --- | --- | --- |
|  | 0.5*x*1 + 0.3*x*2 ≤ 500 | (1) |
| 0.3*x*1 + 0.4*x*2 ≤ 400 | (2) |
| 0.2*x*1 + 0.1*x*2 ≤ 200 | (3) |

The funds available for production include both Rs. 3,00,000 cash that the firm possesses and any borrowed funds maximum up to Rs. 2,00,000. Consequently production is limited to the extent that funds are available to pay for production costs. The constraint expressing this relationship is

|  |  |
| --- | --- |
|  | Funds required for production ≤ Funds available |
| i.e. 10*x*1 + 8*x*2 ≤ Rs. 3,00,000 + *x*3 |
| or 10*x*1 + 8*x*2 – *x*3 ≤ Rs. 3,00,000 | (4) |

The borrowed funds constraint (form condition (iii) of the Question) is

|  |  |  |
| --- | --- | --- |
|  | *x*3 ≤ Rs. 2,00,000 | (5) |

The constraint based on the acid-test condition is developed as follows:

|  |  |
| --- | --- |
|  | or 3,00,000 + *x*3 + 4*x*1 + 3*x*2 ≥ (*x*3 + 0.05*x*3) |
| or – 4*x*1 – 3*x*2 + 0.05*x*3 ≤ 3,00,000 | (6) |

Thus, the linear programming problem is given by

|  |  |  |
| --- | --- | --- |
|  | Maximise Z = 4*x*1 + 3*x*2 *–* 0.05*x*3 |  |
| Subject to 0.5*x*1 + 0.3*x*2 ≤ 500 | (Maximum Hours in Deptt. A) |
| 0.3*x*1 + 0.4*x*2 ≤ 400 | (Maximum Hours in Deptt. B) |
| 0.2*x*1 + 0.1*x*2 ≤ 200 | (Maximum Hours in Deptt. C) |
| 10*x*1 + 8*x*2 – *x*3 ≤ Rs. 3,00,000 | (Total Funds Constraint) |
| *x*3 ≤ Rs. 2,00,000 | (Maximum Bank Loan) |
| –4*x*1 – 3*x*2 + 0.05*x*3 ≤ Rs. 3,00,000 | (Acid Test Radio Constraint) |
| where, *x*1, *x*2, *x*3 ≥ 0. | (Non-Negativity Constraint) |

Problem 1.24

WELLTYPE Manufacturing Company produces three types of typewriters; Manual typewriters, Electronic typewriters, and Deluxe Electronic typewriters. All the three models are required to be machined first and then assembled. The time required for the various models are as follows:

|  |  |  |
| --- | --- | --- |
| Types | *Machine Time (in hours)* | *Assembly Time (in hours)* |
| Manual Typewriter | 15 | 4 |
| Electronic Typewriter | 12 | 3 |
| Deluxe Electronic Typewriter | 14 | 5 |

The total available machine time and assembly time are 3,000 hours and 1,200 hours respectively. The data regarding the selling price and variable costs for the three types are:

The company sells all the three types on credit basis, but will collect the amounts on the first of next month. The labour, material and other variable expenses will have to be paid in cash. This company has taken a loan of Rs. 40,000 from a co-operative bank and this company will have to repay it to the bank on 1st April, 20X1. The TNC Bank from whom this company has borrowed Rs. 60,000 has expressed its approval to renew the loan.

The Balance Sheet of this Company as on 31.3.20X1 is as follows:

The company will have to pay a sum of Rs. 10,000 towards the salary of top management executives and other fixed overheads for the month. Interest on long term loans is to be paid every month at 24% per annum. Interest on loans from TNC and Cooperative Banks may be taken to be Rs. 1,200 for the month. Also, this company has promised to deliver 2 Manual typewriters and 8 Deluxe Electronic typewriters to one of its valued customers next month. Also make sure that the level of operations in this company is subject to the availability of cash next month. This company will also be able to sell all their types of typewriters in the market. The Senior Manager of this company desires to know as to how many units of each typewriter must be manufactured in the factory next month so as to maximise the profits of the company. Formulate this as a linear programming problem. The formulated problem need not be solved.

Solution

Let *x*1, *x*2 and *x*3 denote the number of Manual, Electronic and Deluxe Electronic typewriters respectively to be manufactured in the factory next month.

**Calculation of Contribution Per Unit**

**Calculation of Cash Available for Production**

The objective of the company is to maximise the profit, hence the objective function is given by —

|  |  |  |
| --- | --- | --- |
|  | Maximise Z = 1600*x*1 + 3,000*x*2 + 5,600*x*3 | |
| Subject to the constraints: | |
| 15*x*1 + 12*x*2 + 14*x*3 ≤ 3,000 | (Maximum Machine Hours Constraint) |
| 4*x*1 + 3*x*2 + 5*x*3 ≤ 1,200 | (Maximum Assembly Hours Constraint) |
| 2,500*x*1 + 4,500*x*2 + 9,000*x*3 ≤ 1,36,800 | (Maximum Funds Constraint) |
| *x*1 ≥ 2      (Minimum Supply of Manual Type Constraint) | |
| *x*3 ≥ 8      (Minimum Supply of Deluxe etc. Type Constraint) | |
| *x*2 ≥ 0      (Non-Negativity Constraint) | |

Problem 1.25

A company is interested in the analysis of two products which can be made from the idle time of labour and machine. It was found on investigation that the labour requirement for the first and the second products was 2 and 3 hours respectively and the total available man hours was 24. Only product 1 requires machine hour utilization of one hour per unit and at present only 9 spare machine hours are available. Product 2 requires one unit of a by-product per unit and the daily availability of the by-product is 6 units. According to the marketing department the sales potential of product 1 cannot exceed 5 units. In a competitive market, product 1 can be sold at a profit of Rs. 3 and product 2 at a profit of Rs. 5 per unit.

*Required:* Formulate the problem as a linear programming problem.

Solution

|  |  |  |
| --- | --- | --- |
| Let | *x*1 = Product 1, *x*2 = Product 2 | |
| Since the objective is to maximise the profit, the objective function is given by - | |
| Maximise Z = 3*x*1 + 5*x*2 | |
| Subject to: | |
| 2*x*1 + 3*x*1 ≤ 24 | (Maximum Man Hours constraint) |
| *x*1 ≤ 9 | (Maximum Machine Hours constraint) |
| *x*2 ≤ 6 | (Maximum By-Product constraint) |
| *x*1 ≤ 5 | (Maximum Sales constraint) |
| *x*1, *x*2 ≥ 0 | (Non-Negativity constraint) |

Problem 1.26

A farmer has 2,000 acres of land on which he can grow corn, wheat or soyabeans. Each acre of corn costs Rs. 2,000 for preparation, requires 7 man-days of work and yields a profit of Rs. 600. An acre of wheat costs Rs. 2,400 for preparation, requires 10 man-days of work and yields a profit of Rs. 800. An acre of soyabeans costs Rs. 1400 to prepare, requires 8 man days of work and yields a profit of Rs. 400. If the farmer has Rs. 2,00,000 for preparation and can count on 16000 man-days of work, how many acres should be the allocated to each crop to maximise profits? Formulate an LP model.

Solution

|  |  |  |
| --- | --- | --- |
| Let *x*1, *x*2 and *x*3 be the average of corn, wheat and soyabeans respectively. | | |
| Since the objective is to maximize profits, the objective function is given by — | | |
| Maximise Z = | 600*x*1 + 800*x*2 + 400*x*3 | |
| Subject to | 2000*x*1 + 2400*x*2 + 1400*x*3 ≤ 2,00,000 | (Total Funds constraint) |
|  | 7*x*1 + 10*x*2 + 8*x*3 ≤ 16,000 | (Maximum Man Days constraint) |
|  | *x*1 + *x*2 + *x*3 ≤ 2,000 | (Maximum Area constraint) |
|  | *x*1, *x*2, *x*3 ≥ 0 | (Non-Negativity constraint) |

Problem 1.27

An agriculturist has a farm with 125 acres. He produces Radish, Mutter and Potato. Whatever he raises is fully sold in the market. He gets Rs. 5 for Radish per kg, Rs. 4 for Mutter per kg, and Rs. 5 for Potato per kg. The average yield is 1,500 kg of Radish per acre, 1,800 kg of Mutter per acre and 1,200 kg of Potato per acre. To produce each 100 kg of Radish and Mutter and to produce each 80 kg of Potato, a sum of Rs. 12.50 has to be used for manure. Labour required for each acre to raise the crop is 6 man days for Radish and Potato each and 5 man days for Mutter. A total of 500 man days of labour at a rate of Rs. 40 per man day are available.

*Required:* Formulate this as Linear Programming model to maximize the Agriculturist’s total profit.

Solution

**Statement showing Profit Per Acre**

Let *x*1, *x*2 and *x*3 be the number of acres allotted for cultivating radish, mutter and potatoes respectively.

Since, the agriculturist wants to maximise the total profit, hence the objective function of the problem is given by —

Maximise Z = 7072.5*x*1 + 6775*x*2 + 5572.5*x*3

Subject to following constraints:

|  |  |  |
| --- | --- | --- |
|  | *x*1 + *x*2 + *x*3 ≤ 125 | (Land constraint) |
| 6*x*1 + 5*x*2 + 6*x*3 ≤ 500 | (Man-days constraint) |
| *x*1, *x*2 and *x*3 ≥ 0 | (Non-Negativity constraint) |

Problem 1.28

A firm produces three products A, B and C. It uses two types of raw materials I and II of which 5,000 and 7,500 units respectively are available. The raw material requirements per unit of the products are given below:

The labour time for each unit of product A is twice that of product B and three times that of product C. The entire labour force of the firm can produce the equivalent of 3,000 units. The minimum demand of the three products is 600, 650 and 500 units respectively. Also the ratios of the number of units produced must be equal to 2 : 3 : 4. Assuming the profits per unit of A, B and C as Rs. 50, 50 and 80 respectively.

*Required:* Formulate the problem as linear programming model in order to determine the number of units of each product which will maximize the profit.

Solution

Let *x*1, *x*2 and *x*3 be the number of units of product A, B and C respectively.

Let us first formulate labour time constraint

*Multiplying the above equation by* 6

or, 6*x*1 + 3*x*2 + 2*x*3 ≤ 18000

Let us now formulate ratio of no. of units constraint

*x*1 =  (*x*1 + *x*2 + *x*3)

9*x*1 = 2*x*1 + 2*x*2 + 2*x*3

7*x*1 – 2*x*2 – 2*x*3 = 0

*x*2 =  (*x*1 + *x*2 + *x*3)

9*x*2 = 3*x*1 + 3*x*2 + 3*x*3

–3*x*1 + 6*x*2 –3*x*3 = 0

–*x*1 + 2*x*2 – *x*3 = 0

*x*3 =  [*x*1 + *x*2 + *x*3]

9*x*3 = 4*x*1 + 4*x*2 + 4*x*3

– 4*x*1 – 4*x*2 + 5*x*3 = 0

Since the objective is to maximize the profit, the objective function is given by —

Maximise Z = 50*x*1 + 50*x*2 + 80*x*3

Subject to the constraints:

|  |  |  |
| --- | --- | --- |
|  | 3*x*1 + 4*x*2 + 5*x*3 ≤ 5000 | (Raw material I constraint) |
| 5*x*1 + 3*x*2 + 5*x*3 ≤ 7500 | (Raw material II constraint) |
| 6*x*1 + 3*x*2 + 2*x*3 ≤ 18000 | (Labour force constraint) |
| 7x1 – 2x2 – 2x3 = 0 | (Ratio of no. of units constraint) |
| – x1 + 2x2 – x3 = 0 | (Ratio of no. of units constraint) |
| – 4x1 – 4x2 + 5x3 = 0 | (Ratio of no. of units constraint) |
| *x*1 ≥ 600, *x*2 ≥ 650 and *x*3 ≥ 500 | (Minimum no. of units constraint) |

Problem 1.29

A firm buys castings of P and Q type of parts and sells them as finished product after machining, boring and polishing. The purchasing cost for castings are Rs. 3 and Rs. 4 each for parts P and Q and selling costs are Rs. 8 and Rs. 10 respectively. The per hour capacity of machines used for machining, boring and polishing for two products is given below:

|  | ***Parts*** | |
| --- | --- | --- |
| Capacity (per hour) | P | Q |
| Machining | 30 | 50 |
| Boring | 30 | 45 |
| Polishing | 45 | 30 |

The running costs for machining, boring and polishing are Rs. 30, Rs. 22.5 and Rs. 22.5 per hour respectively.

*Required:* Formulate the linear programming problem to find out the product mix to maximize the profit.

Solution

**Calculation of Profit per unit of P and Q.**

Let us now formulate capacity constraints

|  |  |  |
| --- | --- | --- |
|  |  | (Machining constraint) |
| or, 50x + 30y ≤ 1500 | (i) |
|  | (Boring constraint) |
| or, 45x + 30y ≤ 1350 | (ii) |
|  | (Polishing constraint) |
| or, 30x + 45y ≤ 1350 | (iii) |

Since the objective is to maximize the profit, the objective function is given by -

Maximise Z = 2.75*x* + 4.15*y*

Subject to the constraints:

|  |  |  |
| --- | --- | --- |
|  | 50*x* + 30*y* ≤ 1,500 | (Machining constraint) |
| 45*x* + 30*y* ≤ 1,350 | (Boring constraint) |
| 30*x* + 45*y* ≤ 1,350 | (Polishing constraint) |
| where *x,* y ≥ 0 | (Non-Negativity constraint) |

Problem 1.30

A confectioner markets three products, all of which require sugar. His average monthly sales, cost of sales and sugar consumption are as follows:

Due to Government restrictions his sugar quota has been reduced to 1,405 kg. per month.

*Required:* Formulate LPP to maximise profit.

Solution

**Statement showing the Contribution per unit**

|  |  |
| --- | --- |
| Let | *x*1 = Sugar to be used in product X |
| *x*2 = Sugar to be used in product Y |
| *x*3 = Sugar to be used in product Z |
| Since the objective is to maximise profit, the objective function is given by — |
| Maximise Z = 8*x*1 + 5*x*2 + 12.50 *x*3 |
| Subject to constraints: |
| *x*1 + *x*2 + *x*3 ≤ 1405 (Maximum sugar quota constraint) |
| *x*1 ≤ 500 (Maximum sugar requirement in product X) |
| *x*2 ≤ 800 (Maximum sugar requirement in product Y) |
| *x*3 ≤ 240 (Maximum sugar requirement in product Z) |
| *x*1, *x*2, *x*3 ≥ 0 (Non-Negativity constraint) |

Problem 1.31

A company manufactures and markets three products *X, Y* and Z. All the three products are from the same set of machines. Production is limited by machine capacity (4,580 Hours). From the data below, formulate LPP to maximise profits:

Solution

**Statement showing the Contribution per unit of each Product**

**Working Note:** *Calculation of variable cost per unit*

|  |  |
| --- | --- |
| Let | *x*1 = units to be produced of Product X |
| *x*2 = units to be produced of Product Y |
| *x*3 = units to be produced of Product Z |
| Since the objective is to maximise profit, the objective function is given by — |
| Maximise Z = 9.75 *x*1 + 9.00 *x*2 + 7.70 *x*3 |
| Subject to constraints: |
| 39*x*1 + 20*x*2 + 28*x*3 ≤ 2,74,800 (Maximum machine time in minutes constraint) |
| *x*2 ≤ 6,900 (Maximum sales of Product Y) |
| *x*1 ≥ 2000(Minimum sales of Product X) |
| *x*3 ≥ 1600 (Minimum sales of Product Z) |
| *x*2 ≥ (Non-Negativity constraint) |

Problem 1.32

The following particulars are extracted from the records of a company

|  |  |  |
| --- | --- | --- |
|  | *Product A* | *Product B* |
| Selling Price per unit | *Rs.* 100 | *Rs.* 120 |
| Material Cost per unit | *Rs.* 10 | *Rs.* 15 |
| Direct Wages cost per unit | *Rs.* 15 | *Rs.* 10 |
| Direct Expenses per unit | *Rs.* 5 | *Rs.* 6 |
| Machine Hours used per unit | *Rs.* 3 | *Rs.* 2 |
| *Overhead Expenses:* |  |  |
| Fixed per unit | *Rs.* 5 | *Rs.* 10 |
| Variable per unit | *Rs.* 15 | *Rs.* 20 |

Direct material cost per kg of material is Rs. 5. Direct wage per hour is Rs. 5.

*Required:* Assuming raw materials as the key factor, availability of which is 10,000 kg and maximum sales potential of each product being 3,500 units, formulate the LPP to find out the product mix which will yield the maximum profit.

Solution

**Statement showing the Calculation of Contribution per unit**

|  |  |
| --- | --- |
| Let | *x*1 = units to be produced of Product A |
| *x*2 = units to be produced of Product B |
| Since the objective is to maximise profit, the objective function is given by — |
| Maximise Z = 55*x*1 + 69*x*2 |
| Subject to constraints: |
| 2*x*1 + 3*x*2 ≤ 10,000 (Maximum Raw material available constraint) |
| *x*1 ≤ 3,500 (Maximum Sales of Product A) |
| *x*2 ≤ 3,500 (Maximum Sales of Product B) |
| *x*1 *x*2 ≥ 0 (Non-Negativity constraint) |

*Note:* The committed fixed cost is not relevant for decision making. It is only the future differential cost (i.e. the cost which differs from one alternative to another) which is relevant for decision making.

Problem 1.33

From the following particulars, formulate LPP to find the most profitable product mix.

All the three products are produced from the same direct material using the same type of machines and labour. Direct labour, which is the key factor, is limited to 18,600 hours.

Solution

**Statement Showing the Contribution per Unit**

|  |  |
| --- | --- |
| Let | *x*1 = Number of units to be produced of Product A |
| *x*2 = Number of units to be produced of Product B |
| *x*3 = Number of units to be produced of Product C |
| Since the objective is to maximise profit, the objective function is given by — |
| Maximise Z = 25*x*1 + 24*x*2 + 22*x*3 |
| Subject to constraints: |
| 4*x*1 + 3*x*2 + 2*x*3 ≤ 18,600 (Maximum Labour Hours Constraint) |
| *x*1 ≤ 4,000 (Maximum Sales of Product A constraint) |
| *x*2 ≤ 5,000 (Maximum Sales of Product B constraint) |
| *x*3 ≤ 1,500 (Maximum Sales of Product C constraint) |
| *x*1 ≥ 1,800 (Minimum Sales of Product A constraint) |
| *x*2 ≥ 3,000 (Minimum Sales of Product B constraint) |
| *x*3 ≥ 1,200 (Minimum Sales of Product C constraint) |

*Note:* The committed fixed cost is not relevant for decision making. It is only the future differential cost (i.e. the cost which differs from one alternative to another) which is relevant for decision making.

Problem 1.34

Taurus Ltd. produces three products — A, B and C, from the same manufacturing facilities. The cost and other details of the three products are as follows:

The processing hours cannot be increased beyond 200 hours per month. Fixed expenses per month (Rs.) 2,76,000.

*Required:* Formulate LPP to compute the most profitable product-mix.

Solution

**Statement showing the Contribution per unit**

|  |  |
| --- | --- |
| Let | *x*1 = Number of units to be produced of Product A |
| *x*2 = Number of units to be produced of Product B |
| *x*3 = Number of units to be produced of Product C |
| Since the objective is to maximise profit, the objective function is given by — |
| Maximise Z = 80*x*1 + 40*x*2 + 60*x*3 |
| Subject to constraints: |
| (Maximum Processing hours) |
| *x*1 ≤ 2,000 (Maximum Sales of Product A constraint) |
| *x*2 ≤ 4,000 (Maximum Sales of Product B constraint) |
| *x*3 ≤ 2,400 (Maximum Sales of Product C constraint) |
| *x*1, *x*2, *x*3 ≥ 0 (Non-Negativity constraint) |

*Note:* The committed fixed cost is not relevant for decision making. It is only the future differential cost (i.e. the cost which differs from one alternative to another) which is relevant for decision making.

Problem 1.35

Vinak Ltd. which produces three products furnishes you the following data for 20X1-20X2

The fixed expenses are estimated at Rs. 6,80,000. The Company uses a single raw material in all the three products. Raw material is in short supply and the Company has a quota for the supply of raw materials of the value of Rs. 18,00,000 for the year 20X1-20X2 for the manufacture of its products to meet its sales demand.

*Required:* Formulate the LPP to Set a product mix which will give a maximum overall profit.

Solution

**Statement showing the Calculation of Contribution per unit**

|  |  |
| --- | --- |
| Let | *x*1 = Number of units to be produced of Product A |
| *x*2 = Number of units to be produced of Product B |
| *x*3 = Number of units to be produced of Product C |
| Since the objective is to maximise profit, the objective function is given by — |
| Maximise Z = 10*x*1 + 15*x*2 + 20*x*3 |
| Subject to constraints: |
| 45*x*1 + 30*x*2 + 15*x*3 ≤ 18,00,000 (Maximum Raw Material available constraint) |
| *x*1 ≤ 40,000 (Maximum Sales of Product A) |
| *x*2 ≤ 25,000 (Maximum Sales of Product B) |
| *x*3 ≤ 10,000 (Maximum Sales of Product C) |
| *x*1, *x*2, *x*3 ≥ 0 (Non-Negativity constraint) |

*Note:* The committed fixed cost is not relevant for decision making. It is only the future differential cost (i.e. the cost which differs from one alternative to another) which is relevant for decision making.

Problem 1.36

A firm can produce three different products from the same raw material using the same production facilities. The requisite labour is available in plenty at Rs. 8 per hour for all products. The supply of raw material, which is imported at Rs. 8 per kg., is limited to 10,400 kgs. for the budget period. The variable overheads are Rs. 5.60 per hour. The fixed overheads are Rs. 50,000. The selling commission is 10% on sales.

(a) From the following information, you are required to formulate LPP to suggest the most suitable sales mix, which will maximise the firm’s profits.

(b) Assume, in above situation, if additional 4,500 kgs of raw material is made available for production, and further production, will result in additional fixed overheads of Rs. 20,000 and 25 percent increase in the rates per hour for labour and variable overheads. Formulate LPP to decide whether the firm should go in for further production.

Solution

*Case (a)* Without considering additional material.

**Calculation of Contribution per unit**

|  |  |
| --- | --- |
| Let | *x*1 = Number of units to be produced of Product X |
| *x*2 = Number of units to be produced of Product Y |
| *x*3 = Number of units to be produced of Product Z |
| Since the objective is to maximise profit, the objective function is given by — |
| Maximise Z = 7.80*x*1 + 5.60*x*2 + 12.60*x*3 |
| Subject to constraints: |
| .7*x*1 + .4*x*2 + 1.5*x*3 ≤ 10,400 (Maximum Raw material available constraint) |
| *x*1 ≤ 8,000 (Maximum Sales of Product X constraint) |
| *x*2 ≤ 6,000 (Maximum Sales constraint of Product Y constraint) |
| *x*3 ≤ 5,000 (Maximum Sales constraint of Product Z constraint) |
| *x*1, *x*2, *x*3 ≥ 0 (Non-Negativity constraint) |

*Case (b)* Considering additional material.

**Calculation of contribution per unit for additional additional**

|  |  |
| --- | --- |
| Let | *x*1 = Number of units to be produced of Product X |
| *x*2 = Number of units to be produced of Product Y |
| *x*3 = Number of units to be produced of Product Z |
| *x*4 = Number of units to be produced of Product X from additional raw material |
| *x*5 = Number of units to be produced of Product Y from additional raw material |
| *x*6 = Number of units to be produced of Product Z from additional raw material |
| Since the objective is to maximise profit, the objective function is given by — |
| Z = 7.80*x*1 + 5.60*x*2 + 12.60*x*3 + 4.40*x*4 – 1.20*x*5 + 7.50*x*6 – 20,000 |
| Subject to constraints: |
| .7*x*1 + .4*x*2 + 1.5*x*3 ≤ 10,400 (Raw material constraint) |
| .7*x*4 + .4*x*5 + 1.5*x*6 ≤ 4,500 (Additional Raw material constraint) |
| *x*1 + *x*4 ≤ 8,000 (Maximum Sales of Product X constraint) |
| *x*2 + *x*5 ≤ 6,000 (Maximum Sales of Product Y constraint) |
| *x*3 + *x*6 ≤ 5,000 (Maximum Sales of Product Z constraint) |
| *x*1, *x*2, *x*3, *x*4, *x*5, *x*6 ≥ 0 (Non-Negativity constraint) |

*Recommendation:* The firm should go in for further production only if the value of Z in case (b) exceeds the value of Z in case (a), otherwise not.

*Note:* Only the different cost (i.e. Rs. 20,000) is relevant for decision making.

Problem 1.37

(a) A firm produces 5 different products from a single raw material. Raw material is available in abundance at Rs. 6 per kg. The labour rate is Rs. 8 per hour for all products. The plant capacity is 21,000 labour hours for the budget period. Production facilities can produce all the products. The factory overhead rate is Rs. 8 per hour, comprising Rs. 5.60 per hour as fixed overhead and Rs. 2.40 per hour as variable overhead. The Selling Commission is 10 percent of the product price. Given the following information, formulate LPP to maximise the Company’s profits.

(b) Assume, in above situation, 3,500 hours of overtime working is possible. It will result in additional fixed overheads of Rs. 20,000, a doubling of labour rates and a 50 percent increase in variable overheads. Formulate LPP to decide about the overtime working.

Solution

*Case (a)*

**(a) Statement Showing the Contribution and Ranking of the Product**

|  |  |
| --- | --- |
| Total fixed overheads = Rs. 21,000 × Rs. 5.60 = Rs. 1,17,600 | |
| Let |  |
| *x*1 = Number of units to be produced of Product A during normal working hours |
| *x*2 = Number of units to be produced of Product B during normal working hours |
| *x*3 = Number of units to be produced of Product C during normal working hours |
| *x*4 = Number of units to be produced of Product D during normal working hours |
| *x*5 = Number of units to be produced of Product E during normal working hours |
| Maximise Z = 14.20*x*1 + 15.68*x*2 + 18.60*x*3 + 13.16*x*4 + 16.04*x*5 |
| Subject to constraints: |
| 1.00*x*1 + .80*x*2 + 1.50*x*3 + 1.10*x*4 + 1.40*x*5 £ 21,000 (Maximum labour hours constraints) |
| *x*1 ≤ 4,000 (Maximum Sales constraint of Product A) |
| *x*2 ≤ 3,600 (Maximum Sales constraint of Product B) |
| *x*3 ≤ 4,500 (Maximum Sales constraint of Product C) |
| *x*4 ≤ 6,000 (Maximum Sales constraint of Product D) |
| *x*5 ≤ 5,000 (Maximum Sales constraint of Product E) |
| *x*1, *x*2, *x*3, *x*4, *x*5 ≥ 0 (Non-Negativity constraint) |

*Case (b)*

**(b) Statement Showing the Contribution after Considering Overtime**

Let

*x*6 = Number of additional units to be produced of Product A after considering overtime proposed

*x*7 = Number of additional units to be produced of Product B after considering overtime proposal

*x*8 = Number of additional units to be produced of Product C after considering overtime proposal

*x*9 = Number of additional units to be produced of Product D after considering overtime proposal

*x*10 = Number of additional units to be produced of Product E after considering overtime proposal

Since the objective is to maximise profit, the objective function is given by —

Maximise Z = 14.20*x*1 + 15.68*x*2 + 18.60*x*3 + 13.16*x*4 + 16.04*x*5 + 5.00*x*6 + 8.32*x*7 + 4.80*x*8 + 3.04*x*9 + 3.16*x*10 – 20,000

Subject to constraints:

1.00*x*1 + .80*x*2 + 1.50*x*3 + 1.10*x*4 + 1.40*x*5 ≤ 21,000 (Maximum Normal Hours)

1.00*x*6 + .80*x*7 + 1.50*x*8 + 1.10*x*9 + 1.40*x*10 ≤ 3,500 (Maximum Overtime Hours)

*x*1 + *x*6 ≤ 4,000 (Maximum Sales of Product A constraint)

*x*2 + *x*7 ≤ 3,600 (Maximum Sales of Product B constraint)

*x*3 + *x*8 ≤ 4,500 (Maximum Sales of Product C constraint)

*x*4 + *x*9 ≤ 6,000 (Maximum Sales of Product D constraint)

*x*5 + *x*10 ≤ 5,000 (Maximum Sales of Product E constraint)

*x*1, *x*2, *x*3, *x*4, *x*5, *x*6, *x*7, *x*8, *x*9, *x*10 ≥ 0 (Non-negativity constraint)

*Recommendation:* The firm should go in for further production only if the value of Z in case (b) exceeds the value of Z in case (a) otherwise not.

*Note:* Only the different cost (i.e. Rs. 20,000) is relevant for decision making.

Problem 1.38

PCT Ltd. provides you the following data for three products:

*Required:* Formulate LPP to maximise the profits if maximum availability of raw material is 7,000 kg.

Solution

**Calculation of Contribution per unit**

|  |  |  |
| --- | --- | --- |
| Let | *x*1 = Number of units to be produced of Product A | |
| *x*2 = Number of units to be produced of Product B | |
| *x*3 = Number of units to be produced of Product C | |
| Since the objective is to maximise profit, the objective function is given by — | |
| Maximise Z = 6.00*x*1 + 5.00*x*2 + 4.00*x*3 | |
| Subject to constraints: | |
| 4*x*1 + 2.5*x*2 + 1.6*x*3 ≤ 7,000 (Maximum raw material available constraint) | |
| *x*1 ≤ 2,000 | (Maximum Sales of Product A) |
| *x*2 ≤ 200 | (Maximum Sales of Product B) |
| *x*3 ≤ 4,000 | (Maximum Sales of Product C) |
| *x*1 ≥ 20 | (Minimum Sales of Product A) |
| *x*2 ≥ 40 | (Minimum Sales of Product B) |
| *x*3 ≥ 0 | (Non-Negativity constraint) |

*Note*: The committed fixed cost is not relevant for decision making. It is only future differential cost which is relevant for decision making.

Problem 1.39

COJIH Ltd. is manufacturing three household products *A, B* and C, and selling them in a comparative market. Details of current demand, selling price and cost structure are given below:

The Company is frequently affected by acute scarcity of raw material and high labour turnover. During the next period it is expected to have the following situations:

1. Raw materials available will be only 12,100 kg.
2. Direct labour hours available will be only 5,000 hrs.

*Required:* Formulate LPP which will maximise the company’s profits.

Solution

**Statement Showing the Contribution per Unit**

|  |  |  |
| --- | --- | --- |
| Let | *x*1 = Number of units to be produced of Product A | |
| *x*2 = Number of units to be produced of Product B | |
| *x*3 = Number of units to be produced of Product C | |
| Since the objective is to maximise profit, the objective function is given by — | |
| Maximise Z = 9*x*1 + 8*x*2 + 5.50*x*3 | |
| Subject to constraints: | |
| 0.6*x*1 + 0.4*x*2 + 0.2*x*3 ≤ 12,100 | (Maximum Raw material available) |
| 0.2*x*1 + 0.2*x*2 + 0.1*x*3 ≤ 5,000 | (Maximum Labour hours available) |
| *x*1 ≤ 10,000 | (Maximum Demand of Product A) |
| *x*2 ≤ 12,000 | (Maximum Demand of Product B) |
| *x*3 ≤ 20,000 | (Maximum Demand of Product C) |
| *x*1, *x*2, *x*3, ≥ 0 | (Non-Negativity constraint) |

*Note:* The committed fixed cost is not relevant for decision making. It is only future differential cost which is relevant for decision making.

Problem 1.40

Tulsian Ltd. provides you the following information:

Maximum Raw Material Available 1,00,000 kg @ Rs. 10 per kg. Max Production Hours Available 1,84,000 @ Re. 0.80 with facility for a further 15,000 hours on overtime basis at twice the normal wage rate. Formulate LPP which will maximise company’s profit.

Solution

**Statement Showing the Contribution per Unit during Normal Hours**

**Calculation of Contribution per Unit during Overtime Hours**

|  |  |
| --- | --- |
| Let |  |
| *x*1 = Units of P1 to be produced during Normal Time | |
| *x*2 = Units of P2 to be produced during Normal Time | |
| *x*3 = Units of P3 to be produced during Normal Time | |
| *x*4 = Units of P1 to be produced during overtime | |
| *x*5 = Units of P2 to be produced during overtime | |
| *x*6 = Units of P3 to be produced during overtime | |
| Since the objective is to maximise profit, the objective function is given by — | |
| Maximise Z = 10*x*1 + 15*x*2 + 30*x*3 *–* 2*x*4 *–* 5*x*5 + 14*x*6 | |
| Subject to constraints: | |
| *x*1 + *x*4 ≤ 6000 | (Maximum Sales of Product P1) |
| *x*2 + *x*5 ≤ 4000 | (Maximum Sales of Product P2) |
| *x*3 + *x*6 ≤ 3000 | (Maximum Sales of Product P3) |
| 15*x*1 + 25*x*2 + 20*x*3 ≤ 1,84,000 | (Maximum Normal Production Hours) |
| 15*x*4 + 25*x*5 + 20*x*6 ≤ 15,000 | (Maximum Overtime Production Hours) |
| 10*x*6 + 6*x*2 + 15*x*3 + 10*x*4 + 6*x*5 + 15*x*6 ≤ 1,00,000 (Maximum Raw Material available) | |
| *x*1, *x*2, *x*3, *x*4, *x*5, *x*6 ≥ 0 | (Non-Negativity constraint)) |

Problem 1.41

M/s PL Agro Ltd., engaged in agricultural activities has 500 hectares of virgin land, which can be used for growing jointly or individually tea, coffee, and cardamom. The yield per hectare of the different crops and their selling prices per kg are as under:

|  |  |  |
| --- | --- | --- |
|  | *Yield (kg)* | *Selling Price (Rs. per kg)* |
| Tea | 2,500 | 25 |
| Coffee | 625 | 50 |
| Cardamom | 125 | 300 |

The relevant cost data are given below:

(a) *Variable cost per kg:*

(b) *Fixed cost per annum:* Cultivation and growing cost Rs. 16,00,000, administrative cost Rs. 4,50,000, land revenue Rs. 2,75,000, repairs and maintenance Rs. 5,00,000, other costs Rs. 6,75,000.

The policy of the company is to produce and sell all the three kinds of products and the maximum and minimum area to be cultivated per product is as follows:

|  |  |  |
| --- | --- | --- |
|  | *Maximum Area (Hectares)* | *Minimum Area (Hectares)* |
| Tea | 320 | 240 |
| Coffee | 100 | 60 |
| Cardamom | 60 | 20 |

*Required:* Formulate LPP to maximise profit.

Solution

**Statement Showing the Contribution per Hectare**

|  |  |  |
| --- | --- | --- |
| Let | *x*1 = Area in hectares to be to be used for production of Tea | |
| *x*2 = Area in hectares to be used for production of Coffee | |
| *x*3 = Area in hectares to be used for production of Cardamom | |
| Since the objective is to maximise profit, the objective function is given by — | |
| Maximise Z = 18,750*x*1 + 21,093.75*x*2 + 14062.50*x*3 | |
| Subject to constraints: | |
| *x*1 + *x*2 + *x*3 ≤ 500 | (Maximum Total area available constraint) |
| *x*1 ≤ 320 | (Maximum Total area available for Tea) |
| *x*2 ≤ 100 | (Maximum Total area available for Coffee) |
| *x*3 ≤ 60 | (Maximum Total area available for Cardamom) |
| *x*1 ≥ 240 | (Minimum Total area to be cultivated for Tea) |
| *x*2 ≥ 60 | (Minimum Total area to be cultivated for Coffee) |
| *x*3 ≥ 20 | (Minimum Total area available for Cardamom) |

*Note:* The committed fixed cost is not relevant for decision making. It is only future differential cost which is relevant for decision making.

Problem 1.42

A farmer owns an orchard which has an area of 350 acres on which he grows apples, apricots, cherries and plums. Of the total areas, 250 acres of land are unsuitable for growing apples & plums and are suitable only for apricots and cherries. On the remaining 100 acres of land, any of the four fruits can be grown.

The marketing policy requires that in each season all the four types of fruits must be produced and quantity of any one type should not be less than 12,000 boxes.

It is also essential that the area devoted to anyone should be in terms of complete acres and not in fraction of an acre. There are no physical or marketing limitations and there is an adequate supply of all types of labour.

The details regarding the selling price, production and costs are given below:

Fixed overheads each season:

Cultivation and Growing Rs. 58,000, Harvesting Rs. 68,000, Transport Rs. 5,000, Administration Rs. 42,000, Land Revenue Rs. 9,000.

*Required:* Formulate LPP to maximise profit.

Solution

**Statement showing the Contribution per acre**

|  |  |  |
| --- | --- | --- |
| Let | *x*1 = Area in hectares to be used for cultivation of Apples | |
| *x*2 = Area in hectares to be used for cultivation of Apricots | |
| *x*3 = Area in hectares to be used for cultivation of Cherries | |
| *x*4 = Area in hectares to be used for cultivation of Plums | |
| Since the objective is to maximise profit, the objective function is given by — | |
| Maximise Z = 3,120*x*1 + 830*x*2 + 1,540*x*3 + 4,570*x*4 | |
| Subject to constraints: | |
| *x*1 + *x*2 + *x*3 + *x*4 ≤ 350   (Maximum area available) | |
| *x*1 + *x*4 ≤ 100 | (Maximum area available for cultivation of Apples & Plums) |
| *x*1 ≥ 24 | (Minimum area to be used for Apples) |
| *x*2 ≥ 80 | (Minimum area to be used for Apricots) |
| *x*3 ≥ 120 | (Minimum area to be used for Cherries) |
| *x*4 ≥ 60 | (Minimum area to be used for Plums) |

*Note:* The committed fixed cost is not relevant for decision making. It is only future differential cost which is relevant for decision making.

Problem 1.43

(a) An engineering company is engaged in producing for products through operations at welding and pressing departments. Products W1 and W2 are produced by welders in the welding department whereas products P1 and P2 are produced by press-operators in the pressing department. Due to specific skill requirements, the welders and press-operators can only work in their own department. The following relevant data are available in respect of the products:

The Company incurs Rs. 50,000 per annum towards fixed costs. The maximum available hours are 20,000 and 16,000 for welding and pressing departments respectively. The demands keep on fluctuating but the minimum demands which are to be met as per management’s decision are 2,000 units of W1, 2,500 units of W2, 1,800 units of PI and 2,200 units of P2. Formulate LPP to maximize the profits.

(b) The production manager suggests that the welders and press-operators can be trained to perform both welding and pressing jobs so that excess demand of any of the products can be met. This decision is going to increase the burden of fixed costs by Rs. 5,000 per annum. Formulate LPP to decide about training of welders and press operators.

Solution

*Case (a)*

**Statement Showing Contribution per Unit**

|  |  |  |
| --- | --- | --- |
| Let | *x*1 = Number of units to be produced of Product W1 | |
| *x*2 = Number of units to be produced of Product W2 | |
| *x*3 = Number of units to be produced of Product P1 | |
| *x*4 = Number of units to be produced of Product P2 | |
| Since the objective is to maximise profit, the objective function is given by — | |
| Maximise Z = 12*x*1 + 10*x*2 + 22*x*3 + 14*x*4 | |
| Subject to constraints: | |
| 4*x*1 + 4*x*2 ≤ 20,000 | (Maximum Welding hours constraint) |
| 5*x*3 + 2*x*4 ≤ 16,000 | (Maximum Pressing hours constraint) |
| *x*1 ≥ 2,000 | (Minimum Sales of Product W1) |
| *x*2 ≥ 2,500 | (Minimum Sales of Product W2) |
| *x*3 ≥ 1,800 | (Minimum Sales of Product P1) |
| *x*4 ≥ 2,200 | (Minimum Sales of Product P2) |

*Case (b)*

|  |  |  |
| --- | --- | --- |
|  | Maximise Z = 12*x*1 + 10*x*2 + 22*x*3 + 14*x*4 *–* 5000 | |
| Subject to constraints: | |
| 4*x*1 + 4*x*2 + 5*x*3 + 2*x*4 ≤ 36,000 | (Maximum Labour hours constraint) |
| *x*1 ≥ 2,000 | (Minimum Sales of Product W1) |
| *x*2 ≥ 2,500 | (Minimum Sales of Product W2) |
| *x*3 ≥ 1,800 | (Minimum Sales of Product P1) |
| *x*4 ≥ 2,200 | (Minimum Sales of Product P2) |

*Recommendation:* The company should go in for training of welders and press operators only if the value of Z in case (b) exceeds the value of Z in case (a) otherwise not.

*Note:* Only the differential cost (i.e. 5,000) is relevant for decision making).

Problem 1.44

A Ltd., operating at 75% level of activity produces and sells two products X and Y. The cost sheets of these two products are as under:

|  |  |  |
| --- | --- | --- |
|  | *Product X* | *Product Y* |
|  | | |
| Units produced and sold | 3,000 | 2,000 |
|  | *Per Unit* | *Per Unit* |
|  | *Rs.* | *Rs.* |
| Direct Materials | 10 | 20 |
|  | | |
| Direct Labour | 20 | 20 |
| Factory Overheads (40% fixed) | 25 | 15 |
| Administration and selling overheads (60% fixed) | 40 | 25 |
| Total cost per unit | 95 | 80 |
| Selling price per unit | 115 | 95 |

Factory overheads are absorbed on the basis of machine hour which is the limiting factor. The machine hour rate is Rs. 10 per hour.

The Company receives an offer from Japan for the purchase of product X at a price of Rs. 87.50 per unit. Alternatively, the Company has another offer from Bangkok for the purchase of product Y at a price of Rs. 77.50 per unit. In both the cases, a special packing charge of Rs. 2.50 per unit has to be borne by the Company. The Company can accept either of the two export orders by utilising the balance of 25% of its capacity.

*Required:* Formulate LPP to maximize the profits.

Solution

**(i) Calculation of Machine Hours Available to Produce Additional Units**

**(ii) Statement showing the Contribution per machine hour**

|  |  |
| --- | --- |
| Let |  |
| *x*1 = Hours to be allocated for product X (offer from Japan) | |
| *x*2 = Hours to be allocated for product Y (offer from Bangkok) | |
| Since the objective is to maximise profit, the objective function is given by — | |
| Maximise Z = 9.6*x*1 + 10.67*x*2 | |
| Subject to constraints: | |
| *x*1 + *x*2 ≤ 3,500 | (Maximum machine hours constraint) |
| *x*1, *x*2 ≥ 0 | (Non-Negativity constraint) |

Problem 1.45

A manufacturer has three products A, B and C. Current sales, cost and selling price details and processing time requirements are as follows:

The firm is working at full capacity (13,500 processing hours per year), fixed manufacturing overheads are absorbed into unit costs by a charge of 200% of variable costs. This procedure fully absorbs the fixed manufacturing overhead. Assuming that:

1. Processing time can be switched from one product line to another.
2. The demand at current selling price is:
3. The selling prices are not to be altered.

*Required:* Formulate LPP to maximize the profits.

Solution

**Statement showing the Contribution per unit**

|  |  |  |
| --- | --- | --- |
| Let | *x*1 = Number of units to be produced of Product A | |
| *x*2 = Number of units to be produced of Product B | |
| *x*3 = Number of units to be produced of Product C | |
| Since the objective is to maximise profit, the objective function is given by — | |
| Maximise Z = 14*x*1 + 23*x*2 + 29*x*3 | |
| Subject to constraints: | |
| *x*1 + *x*2 + 2*x*3 ≤ 13,500 | (Maximum Machine hours) |
| *x*1 ≤ 11,600 | (Maximum Sales of Product A) |
| *x*2 ≤ 8,000 | (Maximum Sales of Product B) |
| *x*3 ≤ 2,000 | (Maximum Sales of Product C) |
| *x*1, *x*2, *x*3, ≥ 0 | (Non-Negativity constraint) |

*Note:* The committed fixed cost is not relevant for decision making. It is only future differential cost which is relevant for decision making.

Problem 1.46

Novelties Ltd. seeks your advice on production mix in respect of the three products Super, Bright and Fine. You have the following information:

From current budget, you have further details as below:

You are also to note that there is a constraint on supply of labour in Department A and its manpower cannot be increased beyond its present level.

*Required:* Formulate LPP to maximize the profits.

Solution

**(i) Calculation of Hours available in Department A, represent the key factor**

**(ii) Calculation of Contribution per Unit**

|  |  |  |
| --- | --- | --- |
| Let | *x*1 = Number of units to be produced of Product Super | |
| *x*2 = Number of units to be produced of Product Bright | |
| *x*3 = Number of units to be produced of Product Fine | |
| Since the objective is to maximise profit, the objective function is given by — | |
| Maximise Z = 144*x*1 + 200*x*2 + 80*x*3 | |
| Subject to constraints: | |
| 6*x*1 + 10*x*2 + 5*x*3 ≤ 1,40,000 | (Maximum Machine hours in Dept. A) |
| *x*1 ≤ 6,000 | (Maximum Sales of Product Super) |
| *x*2 ≤ 8,000 | (Maximum Sales of Product Bright) |
| *x*3 ≤ 12,000 | (Maximum Sales of Product Fine) |
| *x*1, *x*2, *x*3, ≥ 0 | (Non-negativity constraint)) |

*Note:* The committed fixed cost is not relevant for decision making. It is only future differential cost which is relevant for decision making.

Problem 1.47

A company has compiled the following data for preparation of budget for 20X1:

After the Budget was discussed the following action plan was approved for improving the profitability of the Company:

1. Direct labour in Department I which is in short supply should be increased by 15,000 hours by spending fixed overheads of Rs. 8,000 per month.
2. To boost sales, an advertisement programme should be launched at a cost of Rs. 10,000 per month.
3. The selling prices should be reduced by:

A : 2 %        B : 8 %        C : 1%

1. The sales targets have been increased and the sales department has confirmed that the Company will be able to achieve the following quantities of sales:

A: 12,000 Units      B: 6,000 Units      C: 10,000 Units

1. The requirement of direct labour hours of department 2 in excess of 40,000 hours is to be met by overtime working involving double the normal rate.

*Required:* Formulate LPP to give suitable product mix which will maximize profits.

Solution

**(i) Statement Showing the Contribution per Unit**

**(ii) Statement Showing Direct Labour Hour Capacity of Dept. 1**

|  |  |  |
| --- | --- | --- |
| Let | *x*1 = Number of units to be produced of Product A without considering overtime | |
| *x*2 = Number of units to be produced of Product B without considering overtime | |
| *x*3 = Number of units to be produced of Product C without considering overtime | |
| *x*4 = Number of units to be produced of Product D considering overtime | |
| *x*5 = Number of units to be produced of Product E considering overtime | |
| *x*6 = Number of units to be produced of Product F considering overtime | |
| Since the objective is to maximise profit, the objective function is given by — | |
| Maximise Z = 3*x*1 + 8*x*2 + 20*x*3 | |
| Subject to constraints: | |
| *x*1 + 2*x*2 + 4*x*3 + *x*4 + 2*x*5 + 4*x*3 ≤ 55,000 | (Maximum Machine hours in Dept. 1) |
| 2*x*4 + *x*5 + 3*x*6 ≤ 40,000 | (Maximum Normal Machine hours in Dept. 2) |
| *x*4 ≤ 12,000 | (Maximum Sales of Product A) |
| *x*5 ≤ 6,000 | (Maximum Sales of Product B) |
| *x*6 ≤ 10,000 | (Maximum Sales of Product C) |
| *x*1, *x*2, *x*3, ≥ 0 | (Non-Negativity constraint) |

*Note:* The committed fixed cost is not relevant for decision making. It is only the future differential cost which is relevant for decision making.

Problem 1.48

A Company manufactures four products. The cost data per unit are as under:

The fixed costs are estimated at Rs. 2,00,000 per month. The Company employs 250 direct workers, who work eight hours a day for 25 days a month. The direct wage rate is Rs. 6 per hour. It is not possible for the Company to increase its operatives in the short run nor it is practicable to work overtime. The Company’s policy does not allow subcontracting of work.

The Marketing Director has forecast the following demand for a month:

|  |  |
| --- | --- |
| *Product* | *Units* |
| *A* | 5,500 |
| *B* | 5,000 |
| *C* | 6,250 |
| *D* | 8,250 |

The management desires you to revise the product mix in the following manner:

|  |  |
| --- | --- |
| *Case (a)* | to yield the maximum profit for the month. |
| *Case (b)* | in proportion to the quantities forecast by the Marketing Director. |
| *Case (c)* | in proportion to the labour requirements calculated for the forecast of sales of the Marketing Director. |

*Required:* Formulate LPP for each of the three proposals to maximize the profits.

Solution

*Case (a)*

**Statement Showing the Contribution per Unit**

*Working Note: Calculation of Direct Labour Hours per unit*

|  |  |  |
| --- | --- | --- |
| Let | *x*1 = Number of units to be produced of Product A | |
| *x*2 = Number of units to be produced of Product B | |
| *x*3 = Number of units to be produced of Product C | |
| *x*4 = Number of units to be produced of Product D | |
| Since the objective is to maximise profit, the objective function is given by — | |
| Maximise Z = 24*x*1 + 24*x*2 + 15*x*3 + 28*x*4 | |
| Subject to constraints: | |
| 4*x*1 + 3*x*2 + 5*x*3 + 2*x*4 ≤ 50,000 | (Maximum Labour hours available constraint) |
| *x*1 ≤ 5,500 | (Maximum Sales of Product A) |
| *x*2 ≤ 5,000 | (Maximum Sales of Product B) |
| *x*3 ≤ 6,250 | (Maximum Sales of Product C) |
| *x*4 ≤ 8,250 | (Maximum Sales of Product D) |
| *x*1, *x*2, *x*3, *x*4, ≥ 0 | (Non-Negativity constraint) |

*Case (b)* Formulation of Proportionate constraints

*Condition 1*

*x*1 =  (*x*1 + *x*2 + *x*3 + *x*4)

or, 25*x*1 = 5.5*x*1 + 5.5*x*2 + 5.5*x*3 + 5.5*x*4

or, 19.5*x*1 *–* 5.5*x*2 *–* 5.5*x*3 *–* 5.5*x*4 = 0

*Condition 2*

*x*2 =  (*x*1 + *x*2 + *x*3 + *x*4)

or, 25*x*2 = 5*x*1 + 5*x*2 + 5*x*3 + 5*x*4

or, – 5*x*1 + 20*x*2 – 5*x*3 – 5*x*4 = 0

*Condition 3*

*x*3 =  (*x*1 + *x*2 + *x*3 + *x*4)

or, 25*x*3 = 6.25*x*1 + 6.25*x*2 + 6.25*x*3 + 6.25*x*4

or, – 6.25*x*1 – 6.25*x*2 + 18.75*x*3 – 6.25*x*4 = 0

*Condition 4*

*x*4 =  (*x*1 + *x*2 + *x*3 + *x*4)

or, 25*x*4 = 8.25*x*1 + 8.25*x*2 + 8.25*x*3 + 8.25*x*4

or, – 8.25*x*1 – 8.25*x*2 – 8.25*x*3 + 16.75*x*4 = 0

|  |  |  |
| --- | --- | --- |
|  | Since the objective is to maximise profit, the objective function is given by — | |
| Maximise Z = 24*x*1 + 24*x*2 + 15*x*3 + 28*x*4 | |
| Subject to constraints: | |
| 4*x*1 + 3*x*2 + 5*x*3 + 2*x*4 ≤ 50,000 | (Maximum Labour hours constraint) |
| *x*1 ≤ 5,500 | (Maximum Sales of Product A) |
| *x*2 ≤ 5,000 | (Maximum Sales of Product B) |
| *x*3 ≤ 6,250 | (Maximum Sales of Product C) |
| *x*4 ≤ 8,250 | (Maximum Sales of Product D) |
| 19.5*x*1 – 5.5*x*2 *–* 5.5*x*3 *–* 5.5*x*4 = 0 | (Proportionate constraint of Product A) |
| –5*x*1 + 20*x*2 – 5*x*3 – 5*x*4 = 0 | (Proportionate constraint of Product B) |
| –6.25*x*1 – 6.25*x*2 + 18.75*x*3 – 6.25*x*4 = 0 | (Proportionate constraint of Product C) |
| –8.25*x*1 – 8.25*x*2 – 8.25*x*3 + 16.75*x*4 = 0 | (Proportionate constraint of Product D) |
| *x*1, *x*2, *x*3, *x*4 ≥ 0 | (Non-Negativity constraint) |

*Case (c)*

**Statement showing the ratio in proportion to the labour requirement calculated for the forecast of sales**

|  |  |  |
| --- | --- | --- |
|  | *x*1 = .2596 (*x*1 + *x*2 + *x*3 + *x*4) | |
| .7404*x*1 – .2596*x*2 – .2596*x*3 – .2596*x*4 = 0 | |
| *x*2 = .1770 (*x*1 + *x*2 + *x*3 + *x*4) | |
| –.1770*x*1 + .8230*x*2 – .1770*x*3 – .1770*x*4 = 0 | |
| *x*3 = .3687 (*x*1 + *x*2 + *x*3 + *x*4) | |
| –.3687*x*1 – .3687*x*2 + .6313*x*3 – .3687*x*4 = 0 | |
| *x*4 = .1947 (*x*1 + *x*2 + *x*3 + *x*4) | |
| –.1947*x*1 – .1947*x*2 – .1947*x*3 + .8053*x*4 = 0 | |
| Since the objective is to maximise profit, the objective function is given by — | |
| Maximise Z = 24*x*1 + 24*x*2 + 15*x*3 + 28*x*4 | |
| Subject to constraints: | |
| 4*x*1 + 3*x*2 + 5*x*3 + 2*x*4 ≤ 50,000 | (Maximum Labour Hours constraint) |
| *x*1 ≤ 5,500 | (Maximum Sales of Product A) |
| *x*2 ≤ 5,000 | (Maximum Sales of Product B) |
| *x*3 ≤ 6,250 | (Maximum Sales of Product C) |
| *x*4 ≤ 8,250 | (Maximum Sales of Product D) |
| .7404*x*1 – .2596*x*2 – .2596*x*3 – .2596*x*4 = 0 (Proportionate constraint in Product A) | |
| –.1770*x*1 + .8230*x*2 – .1770*x*3 – .1770*x*4 = 0 (Proportionate constraint in Product B) | |
| –.3687*x*1 – .3687*x*2 + .6313*x*3 – .3687*x*4 = 0 (Proportionate constraint in Product C) | |
| –.1947*x*1 – .1947*x*2 – .1947*x*3 + .8053*x*4 = 0 (Proportionate constraint in Product D) | |
| *x*1, *x*2, *x*3, *x*4 ≥ 0 (Non-negativity constraint) | |

*Note:* The committed fixed cost is not relevant for decision making. It is only future differential cost which is relevant for decision making.

Problem 1.49

A company produces four products A, B, C and D which are marketed in cartons. Of the total of 20 machines installed, 8 are suitable for manufacturing all the products and the remaining 12 machines are not suitable for the manufacture of products A and D.

Each machine is in production for 300 days per year and each is used on a given product in terms of full days and not in fractions of days. The Company however, has no problem in obtaining adequate supplies of labour and raw materials.

The marketing policy is that all four products should be sold and the minimum annual production should be 3,000 cartons for each product. Fixed costs budgeted amount to Rs. 50 lacs. Production cost price data are as under:

*Required:* Formulate LPP to maximize the profits.

Solution

**Statement Showing the Contribution Per Machine Day**

|  |  |  |
| --- | --- | --- |
| Let | *x*1 = Machine hours to be utilised for Product A | |
| *x*2 = Machine hours to be utilised for Product B | |
| *x*3 = Machine hours to be utilised for Product C | |
| *x*4 = Machine hours to be utilised for Product D | |
| Since the objective is to maximise profit, the objective function is given by — | |
| Maximise Z = 1,736*x*1 + 416*x*2 + 510*x*3 + 864*x*4 | |
| Subject to constraints: | |
| *x*1 + *x*2 + *x*3 + *x*4 ≤ 6,000 | (Maximum Machine hours) |
| *x*1 + *x*4 ≤ 2,400 | (Maximum Machine hours available for product A and D) |
| *x*1 ≥ 215 | (Minimum Machine Hours requirement for Product A) |
| *x*2 ≥ 750 | (Minimum Machine Hours requirement for Product B) |
| *x*3 ≥ 1,000 | (Minimum Machine Hours requirement for Product C) |
| *x*4 ≥ 500 | (Minimum Machine Hours requirement for Product D) |

Problem 1.50

Something More Ltd. is considering adding to its product line. After a lot of deliberations between the Sales and Production personnel, it is decided that products, P, Q and R would be the most desirable additions to the Company’s product range on account of the technical competency, marketing potential and production flexibility as regards these products. In fact P, Q and R can all be made on the same kind of plant as that is already in use and therefore as regards production, all products can be readily interchanged. However, it is considered necessary to build further plant facilities to cater for additional production.

In this connection the following data are relevant:

It is felt that initially extra plant, facilities can be built to operate at the following five different levels of activity, viz, 1,800,2,300, 2,800, 3,300 and 3,800 machine hours per cost period. The fixed overhead costs for a cost period relevant to these five different levels of activity are estimated at Rs. 15,000, Rs. 20,000, Rs. 26,000, Rs. 33,000 and Rs. 39,000 respectively.

*Required:* Formulate LPP to indicate the level of activity that would seem most desirable to be pursued for such maximization of profits.

Solution

**Statement showing Contribution Per Unit**

|  |  |  |
| --- | --- | --- |
| (i) |  | |
| *x*1 = Number of units to be produced of Product P | |
| *x*2 = Number of units to be produced of Product Q | |
| *x*3 = Number of units to be produced of Product R | |
| Since the objective is to maximise profit, the objective function is given by — | |
| Maximise Z = 150*x*1 + 100*x*2 + 90*x*3 | |
| Subject to constraints: | |
| 15*x*1 + 5*x*2 + 3*x*3 ≤ 1,800 | (Maximum Machine Hours available per cost period) |
| *x*1 ≤ 200 | (Maximum Sales of Product P) |
| *x*2 ≤ 125 | (Maximum Sales of Product Q) |
| *x*3 ≤ 750 | (Maximum Sales of Product R) |
| *x*1, *x*2, *x*3 ≥ 0 | (Non-Negativity constraint) |
| (ii) |  | |
| Maximise Z = 150*x*1 + 100*x*2 + 90*x*3 – 5,000 | |
| Subject to constraints: | |
| 15*x*1 + 5*x*2 + 3*x*3 ≤ 2,300 | (Maximum Machine Hours available per cost period) |
| *x*1 ≤ 200 | (Maximum Sales of Product P) |
| *x*2 ≤ 125 | (Maximum Sales of Product Q) |
| *x*3 ≤ 750 | (Maximum Sales of Product R) |
| *x*1, *x*2, *x*3 ≥ 0 | (Non-Negativity constraint) |
| (iii) |  | |
| Maximise Z = 150*x*1 + 100*x*2 + 90*x*3 – 11,000 | |
| Subject to constraints: | |
| 15*x*1 + 5*x*2 + 3*x*3 ≤ 2,800 | (Maximum Machine Hours available per cost period) |
| *x*1 ≤ 200 | (Maximum Sales of Product P) |
| *x*2 ≤ 125 | (Maximum Sales of Product Q) |
| *x*3 ≤ 750 | (Maximum Sales of Product R) |
| *x*1, *x*2, *x*3 ≥ 0 | (Non-Negativity constraint) |
| (iv) |  | |
| Maximise Z = 150*x*1 + 100*x*2 + 90*x*3 – 18,000 | |
| Subject to constraints: | |
| 15*x*1 + 5*x*2 + 3*x*3 ≤ 3,300 | (Maximum Machine Hours available per cost period) |
| *x*1 ≤ 200 | (Maximum Sales of Product P) |
| *x*2 ≤ 125 | (Maximum Sales of Product Q) |
| *x*3 ≤ 750 | (Maximum Sales of Product R) |
| *x*1, *x*2, *x*3 ≥ 0 | (Non-Negativity constraint) |
| (v) |  | |
| Maximise Z = 150*x*1 + 100*x*2 + 90*x*3 *–* 24,000 | |
| Subject to constraints: | |
| 15*x*1 + 5*x*2 + 3*x*3 ≤ 3,800 | (Maximum Machine Hours available per cost period) |
| *x*1 ≤ 200 | (Maximum Sales of Product P) |
| *x*2 ≤ 125 | (Maximum Sales of Product Q) |
| *x*3 ≤ 750 | (Maximum Sales of Product R) |
| *x*1, *x*2, *x*3 ≥ 0 | (Non-Negativity constraint) |

*Recommendation:* The level of activity under which the value of Z is maximum, should be pursued.

*Note:* The committed fixed cost (i.e. 15,000) is not relevant for decision making. Only the future differential costs being relevant for decision making, has been considered.

Problem 1.51

On a turnover of Rs. 20 crores in 20X1, a large manufacturing company earned a profit of 10% before interest and depreciation which were fixed. The product mix was as under:

Interest and depreciation amounted to Rs. 150 lacs and Rs. 77 lacs respectively.

Due to fluctuations in prices in the International Market, the company anticipates that the cost of raw materials which are imported will increase by 10% during 20X2. The company has been able to secure a licence for the import of raw materials of a value of Rs. 1,023 lacs at 20X2 prices. In order to counteract the increase in costs of raw materials, the company is contemplating to revise its product mix. The market survey report recently prepared indicates that the sales potential of each of the products P, Q and R can be increased up to 30% of the total sales value of 20X1. There is no inventory of finished goods or work in process in both the years.

*Required:* Formulate LPP to maximise the profits.

Solution

*Part (i)*

**(a) Statement Showing Original and Revised Material Cost**

**(b) Statement Showing other Variable Cost**

**(c) Statement showing the Contribution to Material Ratio**

**(d) Statement Showing Maximum Sales Potential of Individual Products**

|  |  | ***(Rs. in lacs)*** |
| --- | --- | --- |
| P | 30% of Rs. 2,000 = | 600 |
| Q | 30% of Rs. 2,000 = | 600 |
| R | 30% of Rs. 2,000 = | 600 |
| S | 40% of Rs. 2,000 = | 800 |

|  |  |  |
| --- | --- | --- |
| Let | *x*1 = Sales in value of Product P | |
| *x*2 = Sales in value of Product Q | |
| *x*3 = Sales in value of Product R | |
| *x*4 = Sales in value of Product S | |
| Since the objective is to maximise profit, the objective function is given by — | |
| Maximise Z = .26*x*1 + .165*x*2 + .35*x*3 + .04*x*4 | |
| Subject to constraints: | |
| .44*x*1 + .385*x*2 + .55*x*3 + .66*x*4 ≤ 1,023 | (Maximum Raw Material available) |
| *x*1 ≤ 600 | (Maximum Sales of Product P) |
| *x*2 ≤ 600 | (Maximum Sales of Product Q) |
| *x*3 ≤ 600 | (Maximum Sales of Product R) |
| *x*4 ≤ 800 | (Maximum Sales of Product S) |
| *x*1, *x*2, *x*3, *x*4 ≥ 0 | (Non-Negativity constraint) |

*Note:* The committed fixed cost is not relevant for decision making. It is only differential cost which is relevant for decision making.

Problem 1.52

The costs per unit of the three products *A, B* and *C* of a company are given below:

Production arrangements are such that if one product is given up, the production of the others can be raised by 50%.

*Required:* Formulate LPP to maximize the profits.

Solution

**Statement Showing the Productwise Contribution**

|  |  |  |
| --- | --- | --- |
| Let | *x*1 = Product A | |
| *x*2 = Product B | |
| *x*3 = Product C | |
| S1, S2, S3 (Slack variables) | |
| Since the objective is to maximise profit, the objective function is given by — | |
| Maximise Z = 3,60,000*x*1 + 1,50,000*x*2 + 1,92,000*x*3 + 0 (S1 + S2 + S3) – 1,22,000 | |
| Subject to constraints: | |
| *x*1 + *x*2 + *x*3 = 2 | (Number of Products constraints) |
| *x*1 + S1 = 1 | (Maximum Product A constraint) |
| *x*2 + S2 = 1 | (Maximum Product B constraint) |
| *x*3 + S3 = 1 | (Maximum Product C constraint) |
| *x*1, *x*2, *x*3, S1, S2, S3 ≥ 0 | (Non-Negativity constraint) |

*Note:* The committed fixed cost is not relevant for decision making. It is only differential cost that is relevant for decision making.